Inequalities

Jensen's inequality states that if g is a convex function, then

$$E[g(X)] \geq g(E[X]).$$

This is often useful in obtaining bounds on stochastic optimization problems.

Another useful bound in RM problems is due to Gallege [200] and involves a bound on the function $(X-x)^+ = \max\{X-x, 0\}$ (the positive part of $X - x$). It states that for any random variable X with mean μ and finite variance σ^2,

$$E[(X-x)^+] \leq \frac{\sqrt{\sigma^2 + (x-\mu)^2} - (x-\mu)}{2}.$$

For example, if X is demand and x is a capacity level, then $(X-x)^+$ is the rejected demand (spilled demand) and the above bound provides an upper bound on the expected spilled demand

Some Useful Distributions

We next provide the basic definitions of the most commonly used distributions in RM problems.

Discrete Distributions

Bernoulli

A random variable X has a Bernoulli distribution if it takes on only two values, 0 and 1. A Bernoulli distribution is characterized by a single parameter q (the probability that $X = 1$) with $0 \leq q \leq 1$. In RM, it is often used as the model of a single cancellation.

The basic definitions and properties are

$$\begin{aligned} P(x) &= \begin{cases} q & x = 1 \\ 1-q & x = 0 \end{cases} \\ E[X] &= q \\ Var(X) &= q(1-q) \\ \psi(s) &= qe^s + (1-q). \end{aligned}$$

Binomial

A random variable X has a binomial distribution if it is the sum of n independent Bernoulli random variables. For example, the number of cancellations in a group of n reservations when each independently cancels with probability q. A binomial distribution is characterized by the two parameters q and n with $0 \leq q \leq 1$ and $n \geq 1$.

The basic definitions and properties are

$$\begin{aligned} P(x) &= \binom{n}{x} q^x (1-q)^{n-x}, \; x = 0, 1, \ldots, n \\ E[X] &= nq \end{aligned}$$